Model Derivation and Control Design of a Buck Converter using

Ansoft Simplorer™

By:

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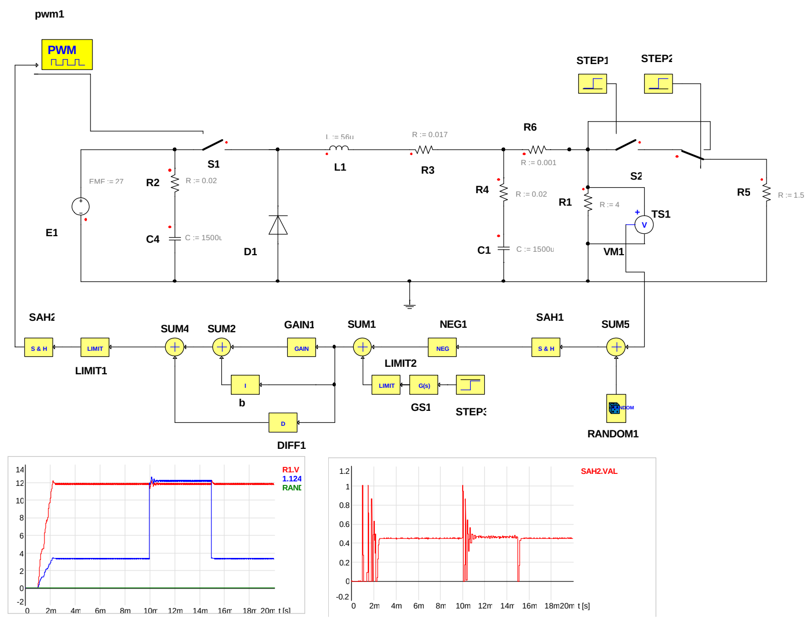
Overview

- Problem Statement
- Buck Theory
- Derive the Transfer Function
- Investigate the Control Methods
- Simulation Results
- Conclusion
- Application: Induction Motor Control
- Questions?

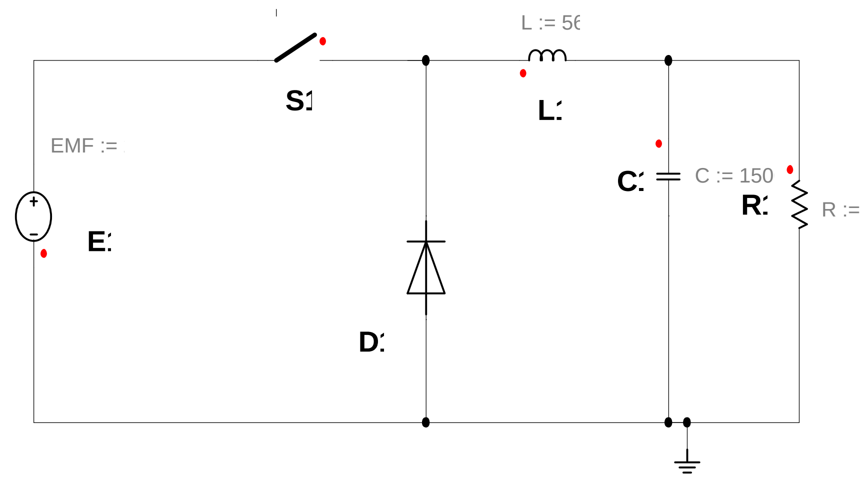
Problem Statement

- Obtain the Mathematical Model of a Buck Converter
- Find a controller that covers the whole range of operation of a Buck Converter including Continuous and Discontinuous Mode



Buck Theory

- Step down the voltage
- Control the Output Voltage while the Load Resistance or the Input Voltage Changes
- Provide Isolation from the Line



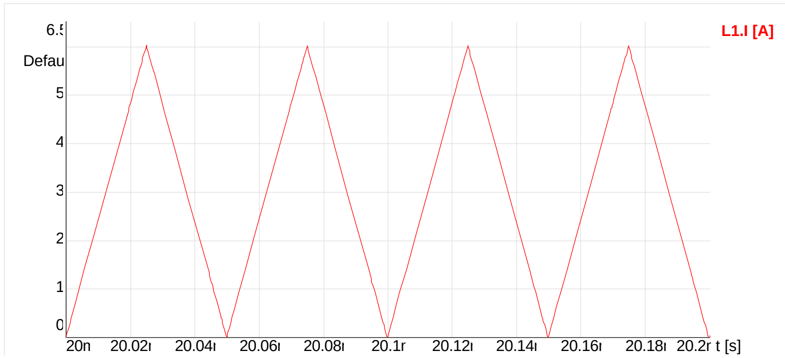
Buck Theory

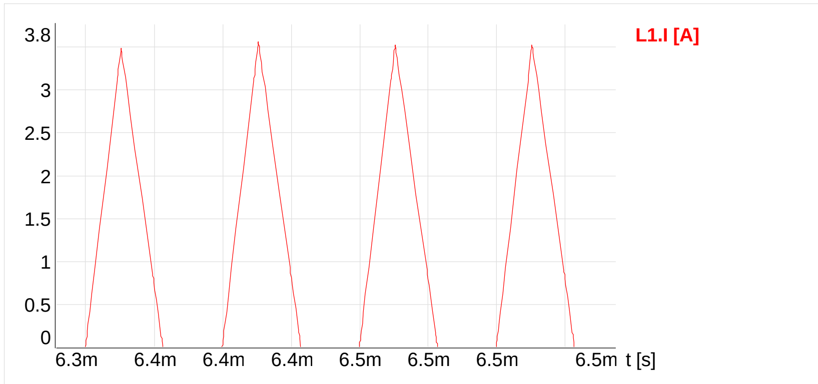
Continuous Conduction Mode

- Inductor Current Stays above zero value for all time

Discontinuous Conduction Mode

- Inductor Current reaches zero value for a while during a one period.

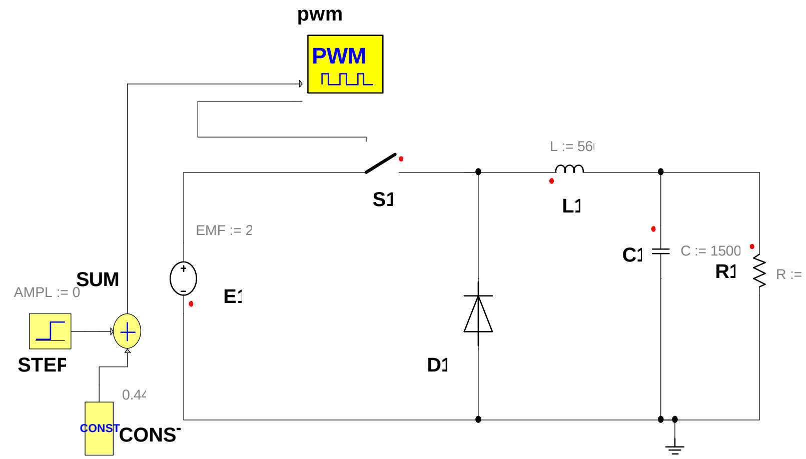




Buck Transfer Function

Continuous Conduction Mode
Simulation Setup

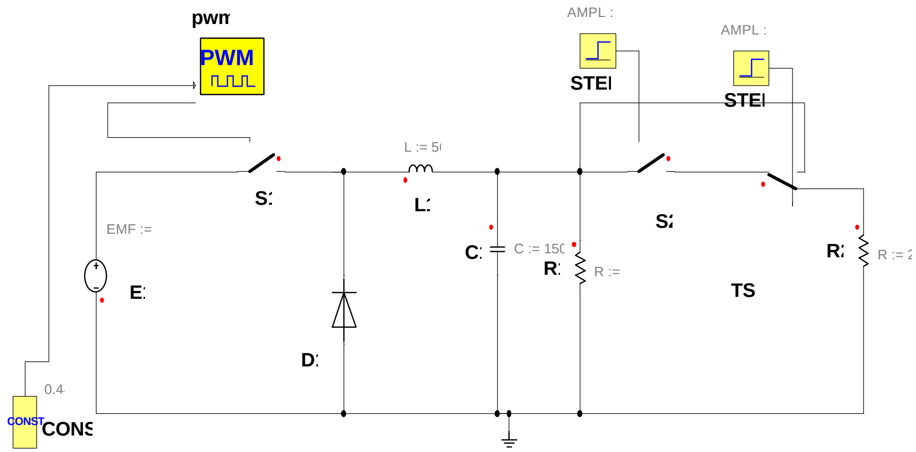
- TF derived from changing the On Time of the Switch resulting in changing the output voltage (CCM)
- Load Resistance changes from 1O, to 1.5O, to 2O, and 3O in CCM



Buck Transfer Function

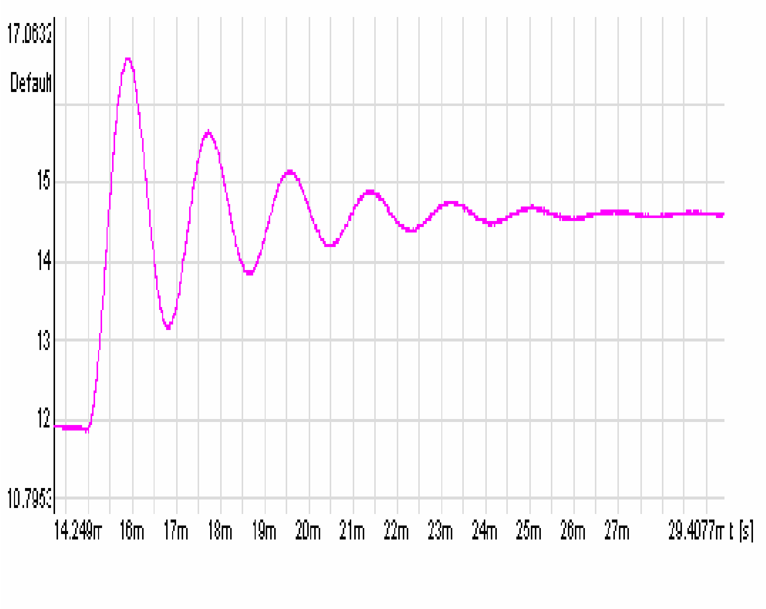
Discontinuous Conduction Mode Simulation Setup

- TF derived from Changing the Load Resistance while keeping the On Time of the PWM Generator Constant (DCM)
- Load changes from 5O, 7O, 10O, and 12O in DCM

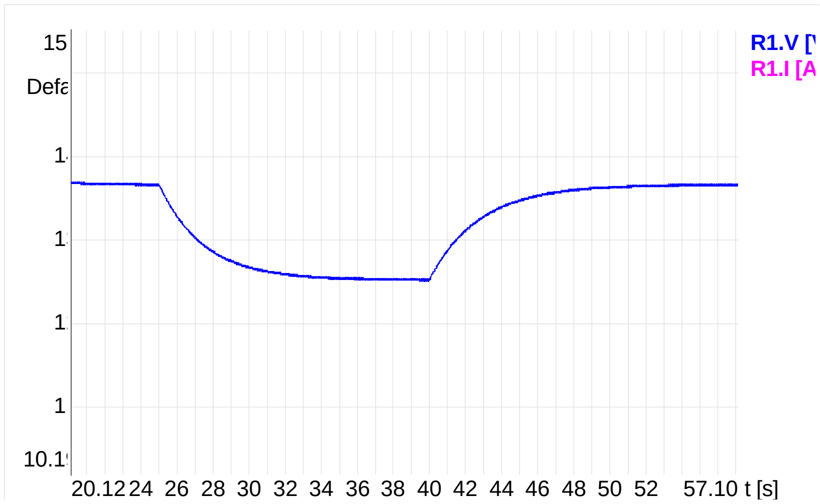


Buck Transfer Function

- Second Order Response for CCM



- First Order Response for DCM



Buck Transfer Function

- Second Order CCM

$V_{Out(IN)ss}$= Output Voltage before adding the change in Duty Ratio

$V_{Out(FV)ss}$= Output Voltage Steady State value after adding the change in the Duty Ratio

$V_{(PEAK)}$= Peak Value of the Overshoot Voltage

$T_{(FV\ First\ Time)}$= Time to the first reach of the Steady State Voltage

$T_{(PEAK)}$= Time to the Peak Voltage

Calculate:

- DC Gain: $K_{DC} = \frac{V_{(FV)ss} - V_{(IN)ss}}{0.1}$
- Ratio of Peak Value to Final Value: $\frac{V_{(PEAK)}}{V_{(Out)ss}}$
- Damping Ratio from Table: ζ=0.55
- Normalized Time to Peak Value: $t^*$from Table
- Calculated Natural Frequency: $w_n = \frac{t^*}{T_{(PEAK)}}$
- Transfer Function:

$$G(s) = \frac{K_{DC} g w_n^{\ 2}}{s^2 + 2 z w_n s + w_n^{\ 2}}$$

$$w_n = 2 \cdot p \cdot f_n$$

$$G(s) = \frac{K_{DC} g (2 \cdot p)^2 \cdot f_n^{\ 2}}{s^2 + 2 \cdot z \cdot \left(2 \cdot p \cdot f_n\right) s + (2 \cdot p)^2 \cdot f_n^{\ 2}}$$

Buck Transfer Function

- First Order DCM

$V_{1(in)}$- Voltage Value before Load Change Occurs

$V_{1(out)}$-Voltage Value after load change occurred and reached steady state

$T_{(63\%)}$-Time it takes for the system to reach 63% of its final value

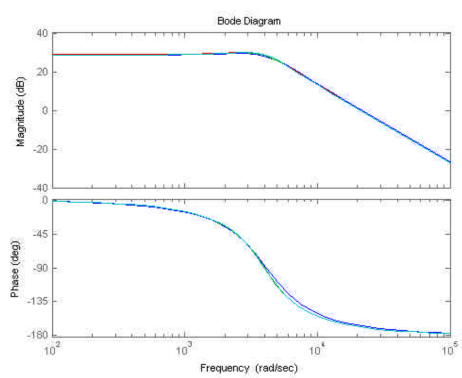
$$H(s) = \frac{K}{t\,s + 1}$$

- K - DC Gain
- t - 63.2% of the final value

Buck Transfer Function

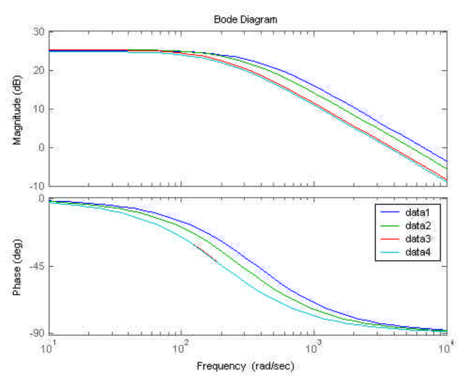
- Continuous Conduction Mode (CCM)
- Discontinuous Conduction Mode (DCM)

$$G(s)=\frac{27\cdot(2\pi)^2\cdot(633)^2}{s^2+2\cdot 0.50\cdot(2\pi)\cdot(633)s+(2\pi)^2\cdot(633)^2}$$



$$G(s)=\frac{K_{DC}.w_c}{s+w_c}$$

•Variable Data	*Test 1*	*Test 2*	*Test 3*	*Test 4*
Resistance (Ω)	5	7	10	12
DC Gain K_{DC}	17.9	18.45	18.4	18.9
Time Constant (Seconds)	0.002705	0.003496	0.004841	0.00485



Investigate Control Methods

- Proportional Integral Derivative (PID)
- Loop Shaping Design (LSD)
- Active Disturbance Rejection Controller (ADRC)

Investigate Control Methods

- Classical PID

$$G_c(s) = k_p \cdot e + k_i \cdot \int e \cdot dt + k_d \cdot \frac{d}{dt} e$$

Investigate Control Methods

- Loop Shaping Design Method (LSD)

$$G_c(s) = \left(\frac{s + w_1}{s} \right)^m \cdot \frac{1}{\frac{s}{w_c} + 1} \cdot \frac{1}{\left(\frac{s}{w_2} + 1 \right)^n} \cdot G_p(s)^{-1}$$

$$G_p(s) = \frac{\frac{1}{C \cdot L}}{s^2 + \frac{1}{C \cdot R} s + \frac{1}{C \cdot L}}$$

Low Pass Filter
Transfer Function

Investigate Control Methods

- Active Disturbance Rejection Controller

$$\ddot{y} = f(t, y(t), \dot{y}(t), w(t)) + b(t) \cdot u(t)$$

$$u = \frac{-z_3 + u_0}{b_0}$$

$$u_0 = k_p.e + k_d.e' \approx k_p.(r - z_1) - k_d.z_2$$

$$k_p = w_c^{\ 2}$$

$$k_d = 2 \cdot w_c$$

Investigate Control Methods

- ADRC State Estimator

$$\begin{Bmatrix} z' = Az + Bu \\ y = Cz \end{Bmatrix}$$ $z_1 \rightarrow y$, $z_2 \rightarrow y'$, and $z_3 \rightarrow f$

$$A = \begin{bmatrix} -3 * w_0 & 1 & 0 \\ -3 * w_0{}^2 & 0 & 1 \\ -3 * w_0{}^3 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 3 * w_0 \\ b_0 & 3 * w_0{}^2 \\ 0 & w_0{}^3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

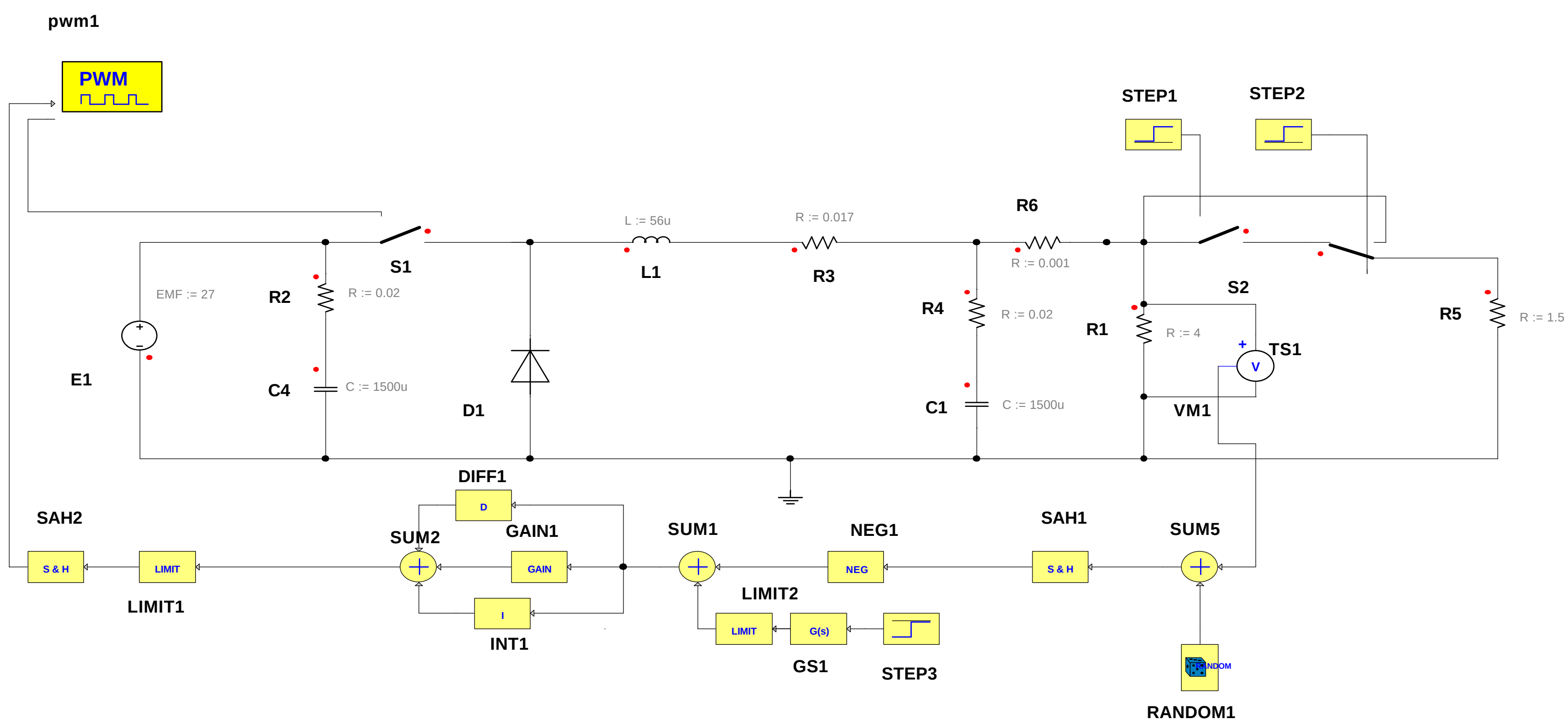
Investigate Control Methods

- Evaluate each Control Method based on a Cost Function of the Form:

$$J = \max(|k_e \cdot e|) + \max(|k_u \cdot u|) + \int_0^T (k_{e2} \cdot e)^2 \, dt + \int_0^T (k_{u2} \cdot u)^2 \, dt$$

Investigate Control Methods

- PID Implementation

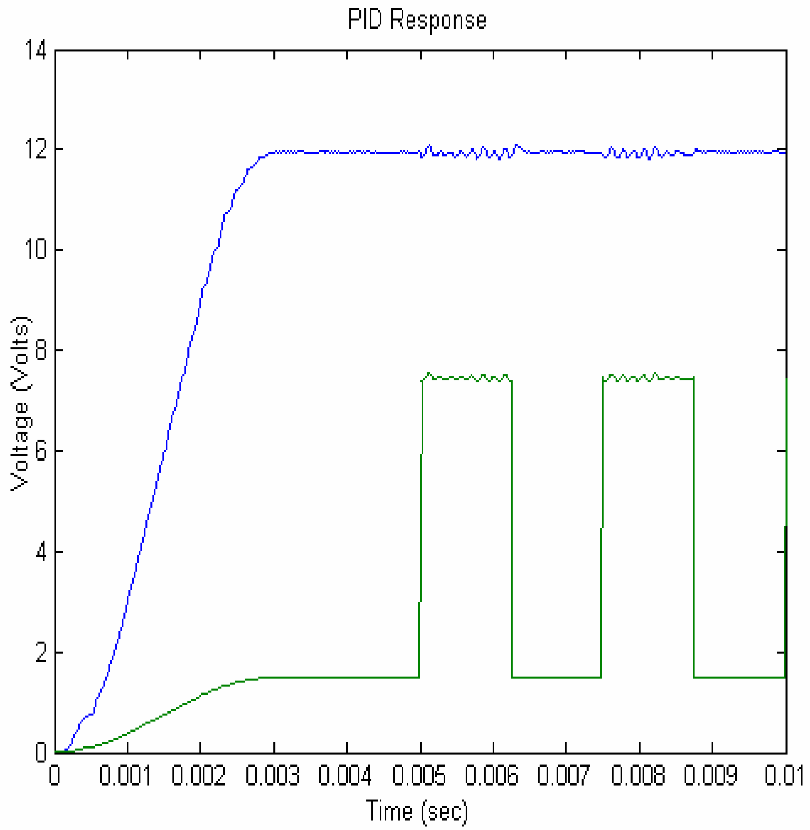


Investigate Control Methods

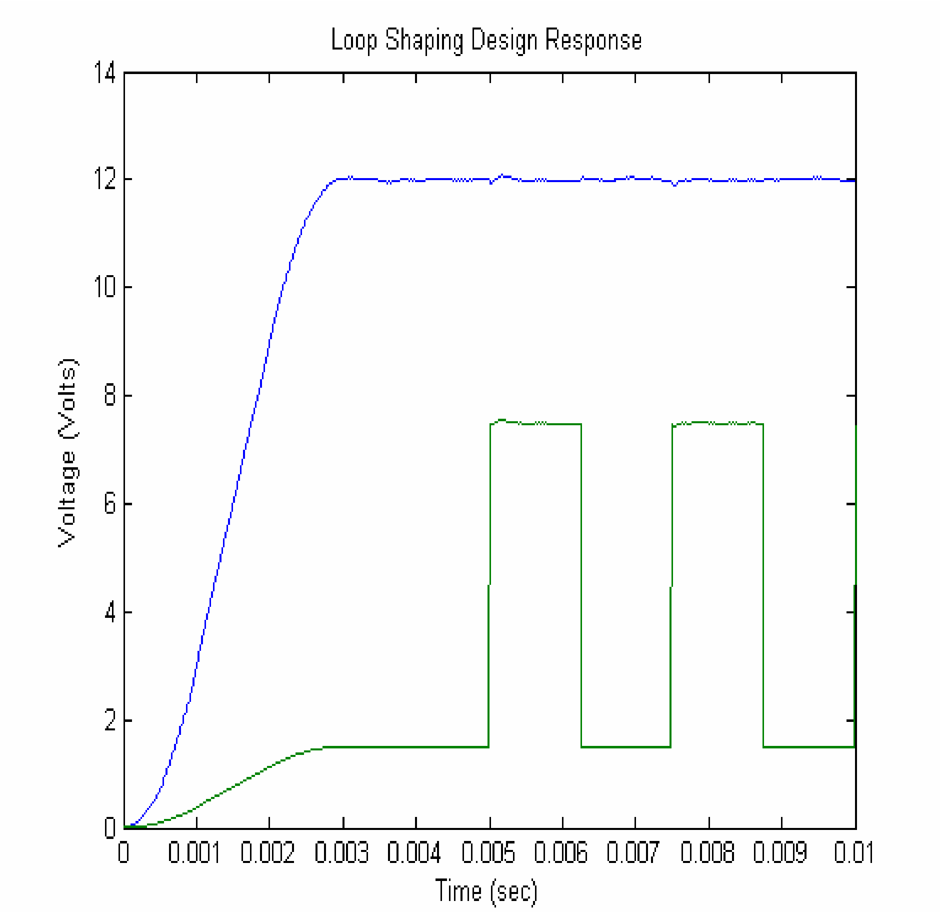
- LSD and ADRC more complex
  - ADRC Require State Estimator
  - LSD easier to implement writing an .m file in Matlab

Simulation Results
PID Response

- Good response for light loads
- Rough response for heavy loads
- Cost Function
  J_{total}=3.976
  E_{max}=1.004
  U_{max}=1.003
  E_{int}=1.053
  U_{int}=0.9171



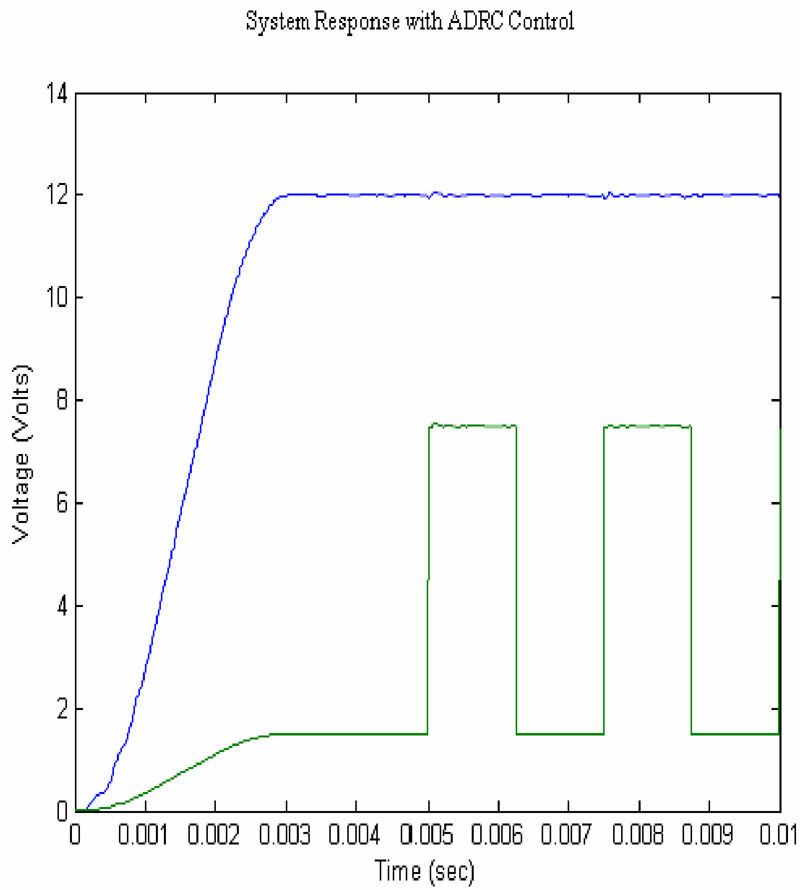
Simulation Results LSD Response



- Good Load Disturbance
- Better than PID
- Cost Function
  J_{total}=3.416
  E_{max}=0.2239
  U_{max}=1.123
  E_{int}=0.1273
  U_{int}=1.942

Simulation Results
ADRC Response

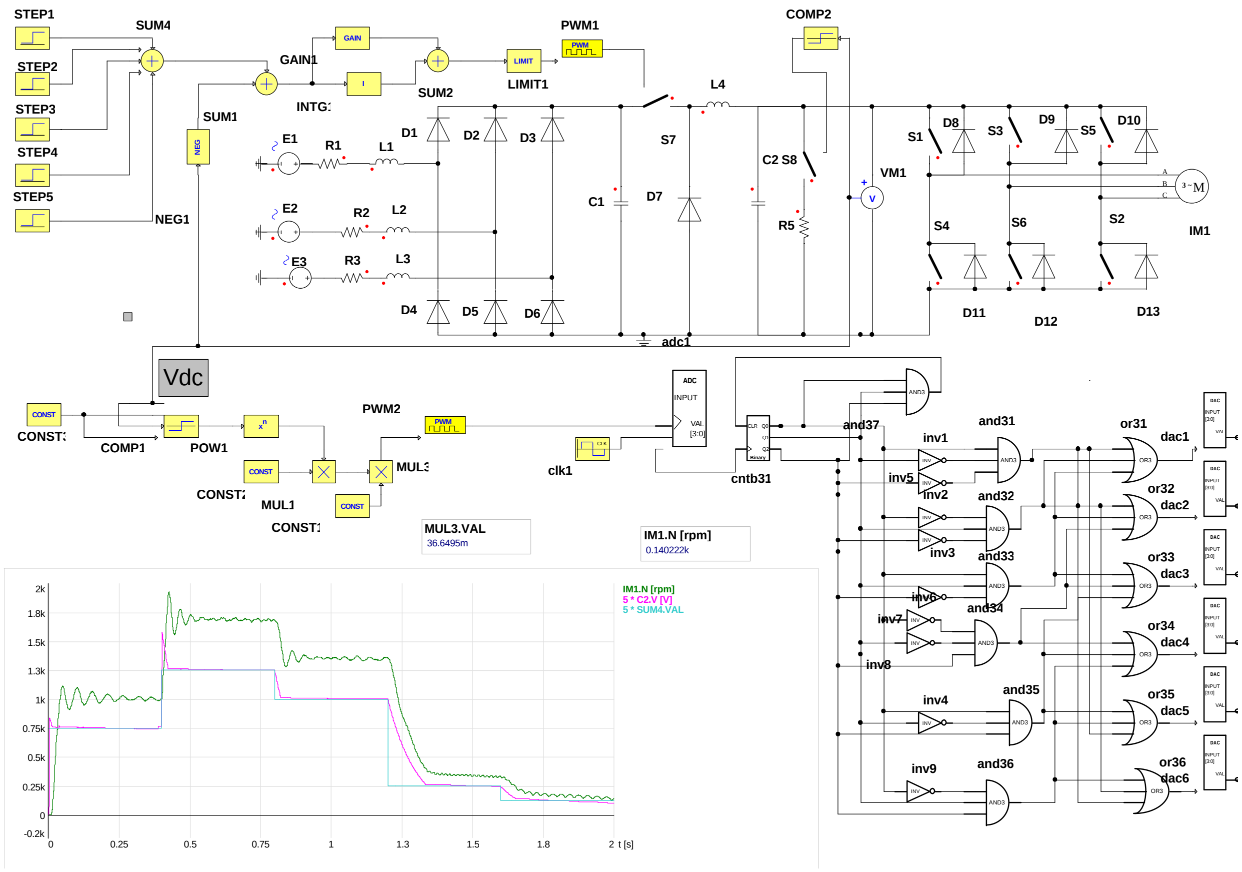
- Good Transient response
- Good Disturbance Rejection
- Cost Function
  J_{total}=10.58
  E_{max}=0.8631
  U_{max}=1.653
  E_{int}=0.5311
  U_{int}=7.533



Conclusion

- ADRC is the preferred Control Method
  Deals Best with Nonlinearities
- LSD has good load disturbance but can not make up for Nonlinearities
- PID lags behind ADRC and LSD
  Not suitable for a Buck Converter.

Buck Application: Induction Motor Control



Questions?